Math 10

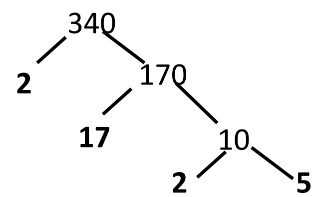
Lesson 1-2 Prime Factorization, GCF and LCM

I. Prime Factorization

Prime factorization is when a number is written as a product of its prime factors.

Example 1 Determine the prime factorization of 340.

There are many ways to solve this problem. One approach is to create a prime factorization tree. Since 340 is even we can remove a factor of 2. 170 remains. Then we can remove 17 from 170 and so on.



∴ the prime factorization of 340 is

$340 = 2 \cdot 2 \cdot 5 \cdot 17$

$= 2^2 \cdot 5 \cdot 17$

In order to do prime factorizations more effectively, it may be helpful to know the following patterns for numbers:

- If a number is even it is divisible by 2.
- If the last two digits of a number are divisible by 4, the whole number is also divisible by 4.
- If a number ends in 5 or 0 it is divisible by 5.
- If the sum of the digits adds up to a number that is divisible by 3, then the original number is also divisible by 3. (For example, consider 123. Since 1 + 2 + 3 = 6 and 6 is divisible by 3, 123 is also divisible by 3. (123 ÷ 3 = 41)

Question 1

Find the prime factorization of 72.

Question 2

Find the prime factorization of 5040.

II. Greatest Common Factor (GCF)

The <u>greatest common factor</u> (GCF) is the largest number that is a factor of all numbers in a given set. For example, what is the GCF of 6 and 15? Note that 6 = 3 x 2 and 15 = 3 x 5. Therefore, the GCF of 6 and 15 is 3.

Example 2 Determine the greatest common factor of 690 and 1518.

There are several ways to solve this problem.

Method A
One approach is to determine the factors of each number and then compare:

1 x 690 = 690	1 x 1518 = 1518
2 x 345 = 690	2 x 759 = 1518
3 x 230 = 690	3 x 506 = 1518
5 x **138** = 690	6 x 253 = 1518
6 x 115 = 690	11 x **138** = 1518
10 x 69 = 690	22 x 69 = 1518
15 x 46 = 690	23 x 66 = 1518
23 x 30 = 690	33 x 46 = 1518

Inspection reveals that **138** is the greatest common factor.

Method B
Write the prime factorization for each number and highlight the factors that appear in each factorization.

690 = **2** · **3** · 5 · **23**
1518 = **2** · **3** · 11 · **23**

The greatest common factor is 2 · 3 · 23 = **138**

Question 3

Find the greatest common factor of 152 and 190.

III. Least Common Multiple (LCM)

The <u>least common multiple</u> (LCM) is the smallest number that is a multiple of all numbers in a given set. For example, what is the least common multiple of 6 and 8?

For 8 we have 8, 16, 24, 32, …

For 6 we have 6, 12, 24, …

∴ the least common multiple of 6 and 8 is 24.

Example 3 Determine the least common multiple of 12, 15 and 40.

Method A

One approach is to list the multiples of each number until the same multiple appears an all 3 lists.

Multiples of 12 are: 12, 24, 36, 48, 60, 72, 84, 96, 108, **120**, …

Multiples of 15 are: 15, 30, 45, 60, 75, 90, 105, **120**, …

Multiples of 40 are: 40, 80, **120**, …

∴ the least common multiple of 12, 15 and 40 is 120.

Method B

Write the prime factorization of each number. Then highlight the <u>greatest</u> power of each prime factor in the list.

$12 = 2^2 \cdot 3$

$15 = 3 \cdot 5$

$40 = 2^3 \cdot 5$

The least common multiple is the product of the greatest power of each prime factor.

$2^3 \cdot 3 \cdot 5 = 120$

∴ the least common multiple of 12, 15 and 40 is **120**.

Question 4

Find the least common multiple of 15, 32 and 44. (Try each method as shown in Example 3.)

Question 5

At Fitz Flooring the Opalescent Arabesque style of tiles measure 20 cm by 36 cm. Assuming the rectangles cannot be cut:

(a) What is the side length of the smallest square that could be tiled? (You may want to check out the example on page 138 in the text book.)

(b) Sketch the square and the rectangular tiles.

(c) Who comes up with the names for styles of tiles?

Question 6

Simplify $\frac{340}{380}$

Question 7

$\frac{9}{14}+\frac{11}{16}=?$

IV. Nasty question of the day

Gold bars in Fort Knox have a proportion of 56 x 28 x 14.

(a) If the bars are stacked into a cube, what is the side length of the smallest cube of gold bars?

(b) How many bars would be in the cube?

(c) What if the proportions were 56 x 29 x 14?

V. Assignment

1. List the first 6 multiples of each number.
   a) 6
   c) 22
   e) 45

2. List the prime factors of each number.
   a) 40
   c) 75
   e) 140

3. Write each number as a product of its prime factors.
   a) 45
   c) 96
   e) 160

4. Use powers (i.e. 2 x 2 x 2 is written as 2^3) to write each number as a product of its prime factors.
   a) 600
   b) 1150
   d) 2250

5. Explain why the numbers 0 and 1 have no prime factors.

6. Determine the greatest common factor of each pair of numbers.
   a) 46, 84
   d) 180, 224

7. Determine the greatest common factor of each set of numbers
   a) 150, 275, 420
   c) 126, 210, 546, 714

8. Determine the least common multiple of each pair of numbers.
   a) 12, 14
   c) 45, 60
   e) 32, 45

9. Determine the least common multiple of each set of numbers.
   a) 20, 36, 38
   c) 12, 18, 25, 30

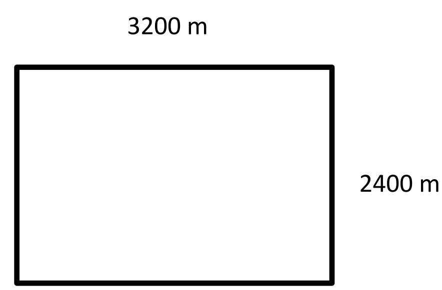
10. Explain the difference between determining the greatest common factor and the least common multiple of 12 and 14.

11. Two marching bands are to be arranged in rectangular arrays with the same number of columns. One band has 42 members, the other has 36 members. What is the greatest number of columns in the array?
12. When is the product of two numbers equal to their least common multiple?

13. How could you use the greatest common factor to simplify a fraction? Use this strategy to simplify these fractions.
    a) $\dfrac{185}{325}$
    c) $\dfrac{650}{900}$
    e) $\dfrac{1225}{2750}$

14. How could you use the least common multiple to add, subtract, or divide fractions? Use this strategy to evaluate these fractions.
    a) $\dfrac{9}{14}+\dfrac{11}{16}$
    c) $\dfrac{5}{24}-\dfrac{1}{22}$
    e) $\dfrac{9}{25}+\dfrac{7}{15}-\dfrac{5}{8}$
    g) $\dfrac{3}{5}\div\dfrac{4}{9}$

15. A developer wants to subdivide this rectangular plot of land into congruent square pieces. What is the side length of the largest possible square?



16. Assuming the tiles cannot be cut:
    a) What are the dimensions of the smallest square that could be tiled using an 18-cm by 24-cm tile?
    b) Could the tiles in part a) be used to cover a floor with dimensions 6.48 m by 15.12 m? Explain.

17. The Dominion Land Survey is used to divide much of western Canada into sections and acres. One acre of land is a rectangle measuring 66 feet by 660 feet.
    a) A section is a square with side length 1 mile. Do the rectangles for 1 acre fit exactly into a section? Justify your answer. [1 mile = 5280 feet]
    b) A quarter-section is a square with side length ½ mile. Do the rectangles for 1 acre fit exactly into a quarter-section? Justify your answer.
    c) What is the side length of the smallest square into which the rectangles for 1 acre will fit exactly?

18. A bar of soap has the shape of a rectangular prism that measures 10 cm by 6 cm by 3 cm. What is the edge length of the smallest cube that could be filled with these soap bars?